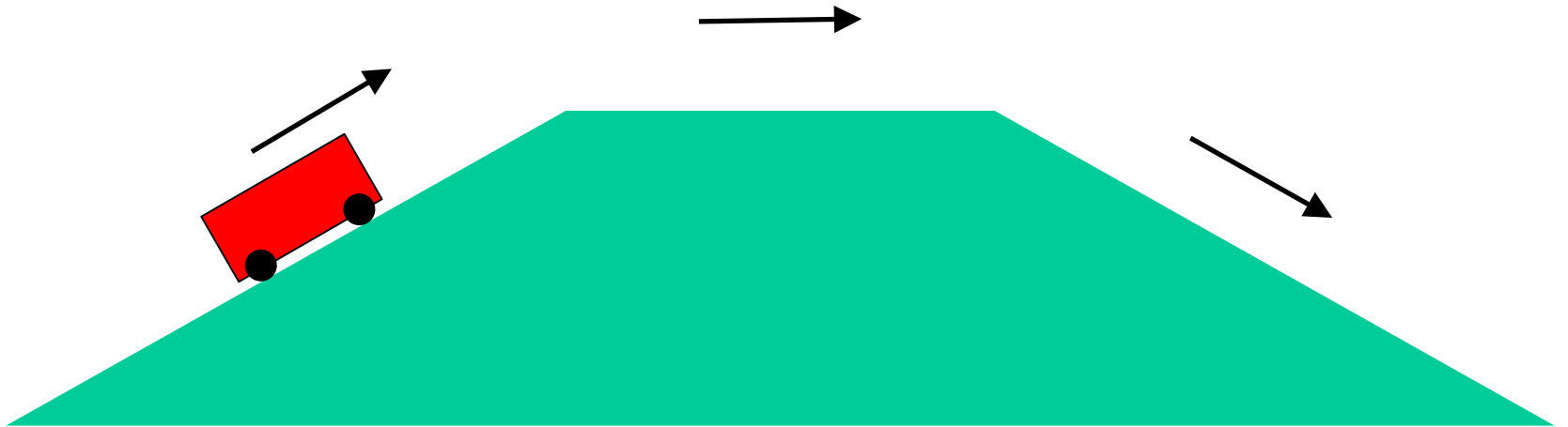


Chapter 10-11: Force, Work, Energy



Work & Kinetic Energy

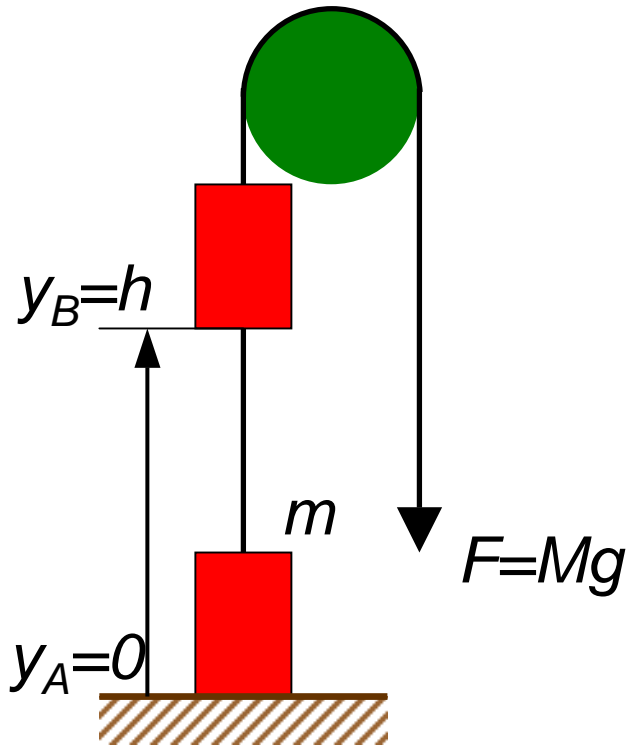
Push a crate with a constant force as indicated.



$$F dx = m v dv = d\left(\frac{1}{2} m v^2\right) = dK$$

$$\text{Work} = \int_A^B F dx = \int_A^B d\left(\frac{1}{2} m v^2\right) = K_B - K_A = \text{Kinetic Energy}$$

Work & Potential Energy



Work: lift a crate from the ground ($y=0$) to a height h .

$$Fdy = mgdy = d(mgy) = dU$$

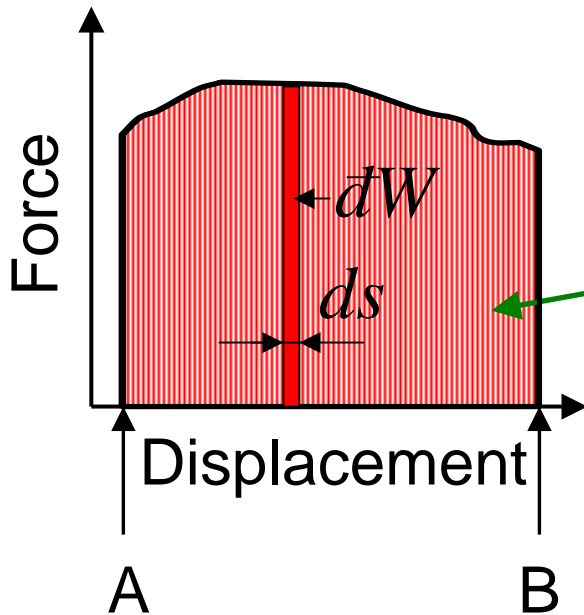
$$\text{Work} = \int_{y_A}^{y_B} Fdy = \int_{y_A}^{y_B} d(Mgy) = U_B - U_A = \text{Potential Energy}$$

A green arrow points from the integral term to the final result, indicating the relationship between the work done and the change in potential energy.

Definition of Work

If the *force* and displacement are in the same direction, work is defined as:

$$dW = F \cdot ds$$



The work is done over a distance by force F :

$$Work = \int_A^B F ds = \text{Area under Force}$$

Properties of Work

(1) Work can be used to change kinetic energy:

$$Work = K_B - K_A = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2$$

(a) *Push: increase kinetic energy*

(b) *Drag: decrease kinetic energy*

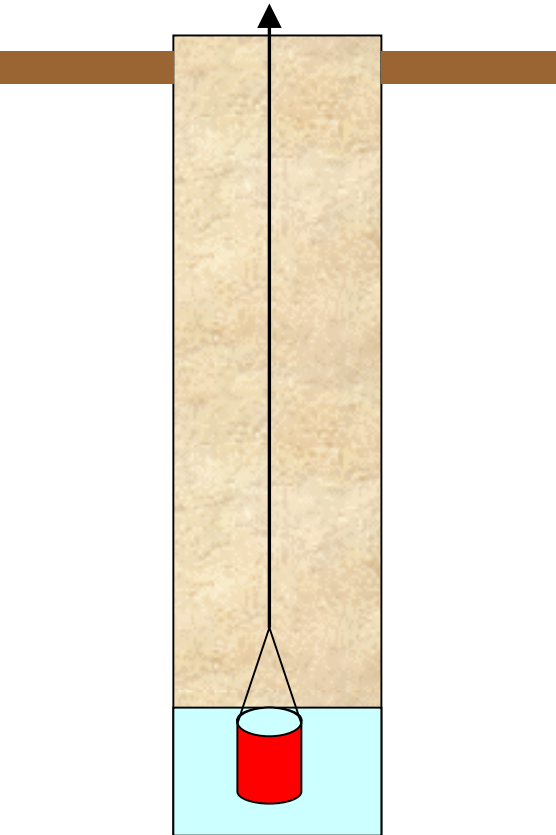
(2) Work can be used to change potential energy:

(a) *Gravitational energy: $Work = U_B - U_A = mg(y_B - y_A)$*

(b) *Elastic energy: $Work = U_B - U_A = \frac{1}{2} \cdot k(\Delta x_B)^2 - \frac{1}{2} \cdot k(\Delta x_A)^2$*

Problem Solving 1

A 10-kg bucket of water is lifted out of a well (5m deep) by a man. The work done by the man is about:



Force:

$$F=mg=10\text{-kg} \times 10\text{m/s}^2=100\text{N}$$

Displacement:

$$\Delta y=5\text{m}$$

Work:

$$W=F \times \Delta y=100\text{N} \cdot 5\text{m}=500\text{J}$$

Problem Solving 2

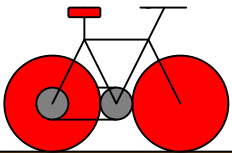
An OSU student rides a bike to campus. The distance from his apartment to the lecture building is 3km (~2 miles). The weight of the student is 75-kg, and that of the bike is 25-kg. The coefficient of kinetic friction is 0.05. What is the work done by the student?

$$\text{Mass: } m=75\text{-kg}+25\text{-kg}=100\text{-kg}$$

$$\text{Force: } F=\mu_k N=0.05\times 100\text{-kg}\times 10\text{m/s}^2=50\text{N}$$

$$\text{Displacement: } \Delta x=3000\text{m}$$

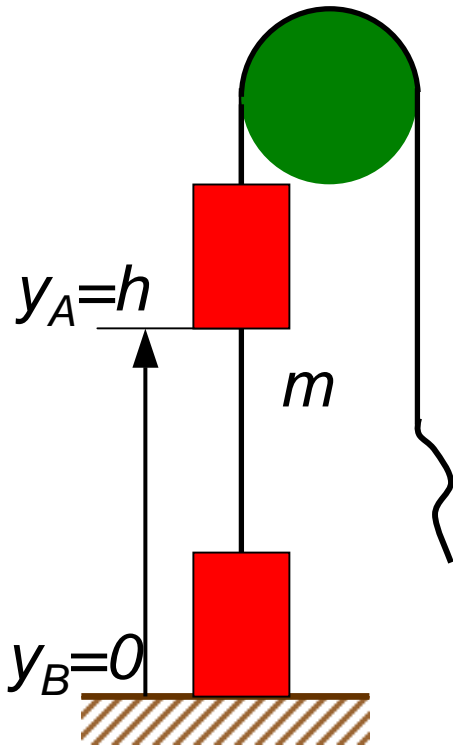
$$\text{Work: } W=F\times\Delta x=50\text{N}\cdot 3000\text{m}=150\text{kJ}$$



Conservative Force

Let a crate drop from a height h to the ground ($y=0$).

$$Fdy = -mgdy = d\left(\frac{1}{2}mv^2\right) = dK$$



$$\int_{y_A}^{y_B} Fdy = U_A - U_B = \int_A^B d\left(\frac{1}{2}mv^2\right) = K_B - K_A$$

Conservative Force Potential Energy Kinetic Energy

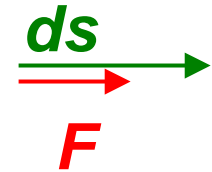
The equation shows the work done by a conservative force F as the crate falls from height y_A to y_B . This work is equal to the change in potential energy, $U_A - U_B$, which is also equal to the change in kinetic energy, $K_B - K_A$. The terms $U_A - U_B$ and $K_B - K_A$ are circled in red, and a green arrow points from the first circle to the second.

General Definition of Work

$$dW = \vec{F} \cdot d\vec{s} = F \times ds \times \cos \theta$$

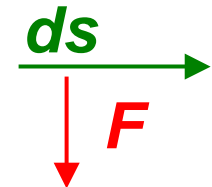
➤ *When F is parallel to the movement:*

$$\theta = 0 \Rightarrow \cos \theta = 1 \Rightarrow dW = F \cdot ds$$



➤ *When F is perpendicular to the movement:*

$$\theta = 90^\circ \Rightarrow \cos \theta = 0 \Rightarrow dW = 0$$



Conservative Forces

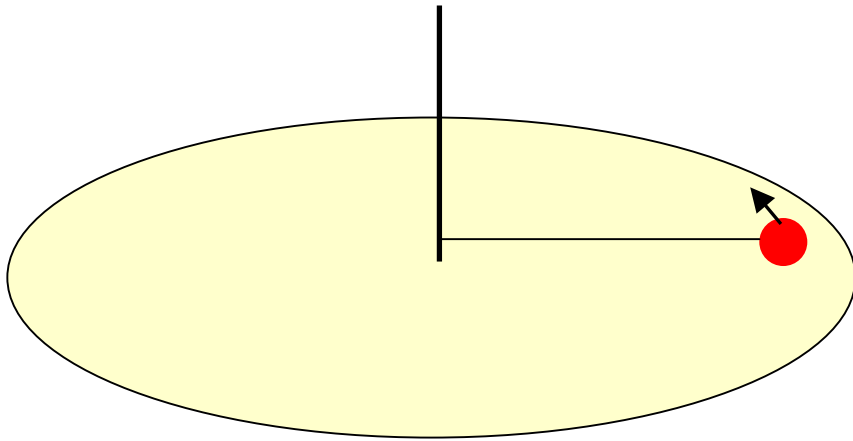
When the total work done by a force F as an object moves over any closed path is zero, the force is conservative.

$$\oint \vec{F} \cdot d\vec{s} = 0$$

- *Gravitational force IS a conservative force*
- *Restoring force (elastic spring) IS a conservative force*
- *Kinetic friction is NOT a conservative force*

Case Study I

A ball lies on a horizontal, frictionless smooth surface, and is tied to a post by a string of length ℓ . The ball is hit by a stick and starts to move in the direction perpendicular to the string. The subsequent motion of the ball is described by:

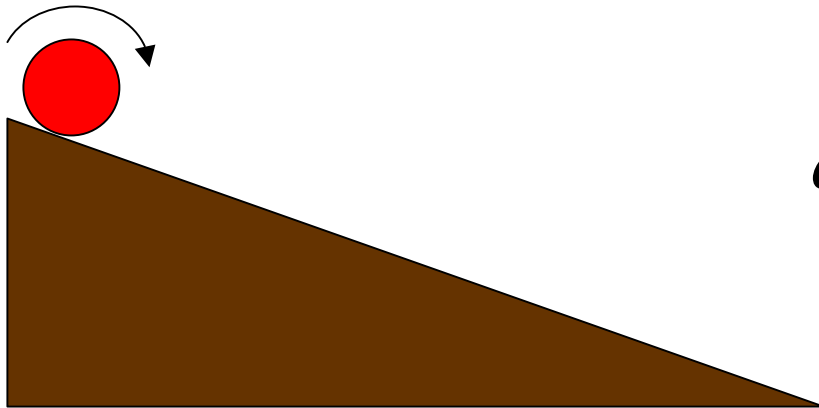


- A. *Circular motion but slow down in time*
- B. *Spiral motion with constant speed*
- C. *Circular motion with constant speed*
- D. *None of above*

$$dW = \vec{F} \cdot d\vec{s} = 0$$

Case Study II

A ball is rolling down a rough surface without sliding. Is the total mechanical energy of the ball conserved?



$$dW = \vec{F} \cdot d\vec{s} = 0$$

Concept Question I

A steel ball moves in a circle at constant speed. The work done by the centripetal force is zero because:

- A. the displacement for each revolution is zero*
- B. the average force for each revolution is zero*
- C. there is no friction*
- D. the centripetal force is perpendicular to the velocity*
- E. the magnitude of the acceleration is zero*

Concept Question II

The work done by gravity during the falling of a ball is :

A. Positive

B. Negative

C. Zero

D. sign depends on the direction of y axis

E. None of above

Summary of Chapters 10 & 11

Kinetic energy: $K = \frac{1}{2}mv^2$

Gravitational potential energy: $U = mgy$

Elastic potential energy: $U = \frac{1}{2}k(\Delta x)^2$

Work: $W = \int_A^B \vec{F} \cdot d\vec{s}$

Conservation of mechanic energy: $E = K + U = \text{constant}$

Conservative force: $\oint \vec{F} \cdot d\vec{s} = 0$

eg: gravitational force, restoring force

Non-conservative force: $\oint \vec{F} \cdot d\vec{s} \neq 0$

eg: kinetic friction, push or pull by a person

Information for PHYS2014 Classes

- HW #7 is due Friday March 28
- HW #8 is due Friday March 28
- Exam #3 may be postponed to April 17 (Thursday)
- Final exam will be on May 1 (Thursday)
- Please visit course website for HW assignments, HW solutions, lecture notes, and other information.
- Please read the textbook before lectures